Circle your Instructor:
Faudree, Williams, Zirbes

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Math 251 Fall 2017 **Quiz #5, October 11th**

Name: Solutions

There are 25 points possible on this quiz. This is a closed book quiz. Calculators and notes are not allowed. **Please show all of your work!** If you have any questions, please raise your hand.

Exercise 1. (6 pts.) Differentiate the following functions.

(a) $f(\theta) = \theta \sin\theta \cos\theta$

$f'(\theta) = 1\cdot(\sin\theta\cos\theta) + \theta \cdot \frac{d}{d\theta}(\sin\theta\cos\theta)$

$= \sin\theta\cos\theta + \theta\cdot[\sin\theta\cdot(-\sin\theta) + \cos\theta\cdot\cos\theta]$

$= \sin\theta\cos\theta + \theta(\cos^2\theta - \sin^2\theta)$

(b) $f(t) = 4^{5t^2} = (4)^{(5t^2)}$

$f'(t) = (\ln 4)\cdot 4^{5t^2}\cdot \frac{d}{dt}[5t^2]$

$= (\ln 4)\, 4^{5t^2}\cdot 10t$

$= (10\ln 4)\, t\cdot 4^{5t^2}$

Exercise 2. (6 pts.) find the derivatives of the following functions.

(a) $g(x) = \sec^5(3x) = [\sec(3x)]^5$

$g'(x) = 5[\sec(3x)]^4 (\sec(3x)\tan(3x))\cdot 3$

($\frac{d}{dx}(\sec(3x))$)

$= 15\sec^5(3x)\tan(3x).$

(b) $f(x) = e^{x\csc x}$

$f'(x) = e^{x\csc x}\cdot \frac{d}{dx}(x\csc x)$

$= e^{x\csc x}\cdot[1\cdot\csc x + x\cdot(-\cot x\csc x)]$

$= \csc x(1 - x\cot x)\, e^{x\csc x}$

Exercise 3. (4 pts.) For what values of x does $y = \sqrt{x^2+5x}$ have a horizontal tangent?

$y = (x^2+5x)^{1/2}$

$y' = \frac{1}{2}(x^2+5x)^{-1/2}(2x+5)$

$= \frac{2x+5}{2\sqrt{x^2+5x}} = 0$

because we know horizontal slope means $y'=0$.

So, $2x+5 = 0$ or $x = -5/2$

But, $x = -5/2$ is not in the domain of y'. (ie The derivative is undefined)

ans: y has no horizontal tangents

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Exercise 4. (4 pts.) Find an equation of the tangent line to the curve $y = \dfrac{8}{(\tan x + 2)^2}$ at the point $(0, 4)$

point

$y = 8(\tan x + 2)^{-2}$

$y' = -16(\tan x + 2)^{-3}(\sec^2 x)$

$= \dfrac{-16\sec^2 x}{(\tan x + 2)^3}$

$y'(0) = \dfrac{-16\sec^2 0}{(\tan 0 + 2)^3} = \dfrac{-16}{8} = -2$ ← slope $= m$

tangent line:

$y - 4 = -2(x - 0)$

$\boxed{y = -2x + 4}$

Correction: The point should have been $(0, 2)$. So the line should have been $y = -2x + 2$.

Exercise 5. (5 pts.) Find the 50th derivative of $y = \sin(2x)$.

(a) Find the first four derivatives of $y = \sin(2x)$.

$y' = 2\cos(2x)$

$y'' = -2^2 \sin(2x)$

$y''' = -2^3 \cos(2x)$

$y^{(4)} = 2^4 \sin(2x)$

(b) Using your answer to (a), find the 50th derivative of $y = \sin(2x)$.

$4\overline{)50}$ → 12, remainder 2

Every 4 derivatives, we get back to $\sin(2x)$.

2 ← 2 derivatives past $4 \cdot 12 = 48$.

Ans: $y^{(50)} = -2^{50}\sin(2x)$